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STRAIN-DEPENDENT CREEP DAMAGE IN RANDOM INHOMOGENEOUS MATERIALS--ETC(U)
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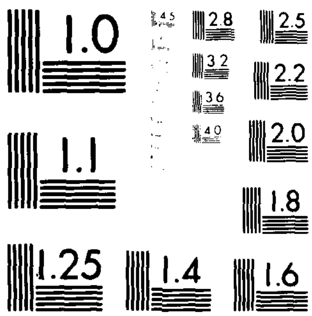

MICROCOPY RESOLUTION TEST CHART

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Department of Engineering Science
FACULTY OF ENGINEERING AND APPLIED SCIENCES

State University of New York at Buffalo

STATE UNIVERSITY OF NEW YORK · BUFFALO ·

Report No. 121

STRAIN-DEPENDENT CREEP DAMAGE IN RANDOM INHOMOGENEOUS MATERIALS*

by

C. Lee and F.A. Cozzarelli

November, 1980

*This research was supported in part by the Office of Naval Research under Contract No. N00014-75-C-0302. Approved for public release. Distribution unlimited.

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ABSTRACT

Employing a recent homogeneous strain-dependent creep damage theory, the propagation of a failure front in a beam under pure bending is studied. A local inhomogeneous strain-dependent creep damage theory is then postulated, based on creep damage data obtained at J.R.C. Ispra for specimens of various lengths. Using this inhomogeneous theory, solutions for various loading conditions are obtained, and are shown to be in better agreement with observation than previous theories. Finally, the effect of random material parameters on the rupture time is considered for the constant tensile stress test and for the beam under pure bending.

LIST OF FIGURES AND TABLES

NOTATION

a, b', c'	damage constants, Equation (63)
a_1, a_2	nondimensional coefficients, Equation (92)
A	instantaneous cross-sectional area
A_o	initial cross-sectional area
A_r	effective undamaged cross-sectional area, Equation (4)
$\tilde{A}$	small area increment, Equation, Equation (90)
$\bar{\tilde{A}}$	nondimensional area, Equation (92)
b, c	damage constants, Equation (64)
$\hat{b}' = \frac{\hat{b}}{b_o}$	normalized random variable
B, ν	material damage constants, Equation (6)
B_o	nondimensional constant, Equation (115)
$C = C_o e^{-\beta/T_o}$	damage constant, Equation (27)
C_o	damage constant, Equation (26)
d	width of the distribution of damage, Equation (72)
D	damage, Equation (28)
D_o	initial damage at t=0, Equation (58)
D_R	experimental critical value of damage
D_T	total damage, Equation (59)
$\tilde{D}$	damage difference, Equation (59)
$\tilde{D}_R$	local critical value of damage
$\tilde{D}_R^{AV}$	average damage at rupture, Equation (62)
E	elastic modulus
E_D	specific dissipated energy, Equation (17)
E^*	fictitious elastic modulus used to approximate transient creep

$f(S), g(S)$	functions of S, Equation (15)
$F(S), G(S)$	functions of S, Equation (14)
h	effective half depth of rectangular beam
h_o	initial effective half depth of rectangular beam
$H(x)$	combined material parameter, Equation (61)
I	nondimensional material constant, Equation (86)
J	property of beam cross-section, Equation (41)
J'	second invariant of stress deviator, Equation (35a)
K, n	creep constants, Equation (2)
K_o, n_o	creep constants for instantaneous deformation, Equation (13)
$\bar{K}$	material rupture constant, Equation (17)
ℓ	length of beam
M	bending moment, Equation (40)
P_o	constant load suddenly applied at $t = 0$
q, r	material powers, Equations (9,10)
Q	constant evaluated at rupture, Equation (62)
R	instantaneous cross-sectional radius
sgn( )	signum function, Equation (38)
S	effective stress
t	time
t_{CR}	rupture time at the stress σ_c
t_K	rupture time at the stress σ_K
t_R	rupture time
t_{RH}	Hoff's ductile rupture time, Equation (3)
t_{RI}	time at initial rupture of beam, Equation (45)
t_{RK}	Kachanov's brittle rupture time, Equation (7)
$t_{R'}$	half rupture time of beam, Equation (57)

T	absolute temperature
T_o	constant temperature
$U(\ )$	unit step function
w	width of rectangular beam
x_o, y_o	initial coordinates of beam, Figure 1
x, y	moving coordinates of beam, Figure 1
$\alpha, \beta, \gamma, \delta$	damage constant, Equation (26)
ε_c	axial creep strain, Equation (1)
ε_ℓ	total lateral strain, Equation (77)
ε_R	creep strain at rupture
$\dot{\varepsilon}_o$	creep strain rate at the stress σ_o
$\dot{\varepsilon}_I, \dot{\varepsilon}_{II}, \dot{\varepsilon}_{III}$	principal strain rates, Equation (20)
$\dot{\varepsilon}_R$	creep strain rate at the reference stress σ_R, Equation (108)
ε_c^*	strain in damage law for multiaxial states of stress, Equation (34)
η, τ	parameters in lognormal probability density function, Equation (102)
θ	constant corresponds to two types of rupture under multiaxial stress, Equation (21)
$\dot{\kappa}$	rate of curvature of beam, Equation (39)
ν_e	elastic Poisson's ratio
ν_s	steady creep Poisson's ratio, Equation (77)
ξ	half thickness of deteriorated region of beam
ρ	density
ρ_o	initial density

σ	one-dimensional tensile stress
σ_o	constant stress
σ_c	particular applied constant tensile stress, Equation (62)
σ_K	constant tensile stress, Equation (18)
σ_{max}	maximum normal tensile stress, Equation (44)
σ_R	reference stress, Equation (108)
$\sigma_I, \sigma_{II}, \sigma_{III}$	principal stresses, Equation (19)
σ^*	stress in damage law for multiaxial states of stress, Equation (34)
ϕ	potential function, Equation (19)
ψ	continuity, Equation (4)
ω	Kachanov material damage parameter, Equation (5)
$( )_o$	indicates nominal value
$(\dot{ })$	indicates d/dt
$( )'$	indicates derivative with respect to agrument
$(\hat{ })$	designates random variable

1. INTRODUCTION

With the development of heat resistant metallic alloys, it is now possible to increase the operating temperatures in thermal machines (e.g., jet turbines and nuclear reactors) and thereby obtain better thermodynamic efficiencies. At these elevated operating temperatures the phenomenon of creep is often very significant, and in particular the problem of creep rupture is of major concern when considering the safety of such thermal machines. Creep rupture in even a small structural component of a nuclear reactor may not only jeopardize the operation of the entire system, but can also result in a serious hazard to the surroundings. With the availability of high speed computers and modern numerical techniques the designer may now obtain accurate creep rupture analyses; this accuracy is primarily limited by the reliability of the creep rupture theory employed. The present research is directed toward the development of improved creep rupture theories, through the inclusion of the effects of strain, inhomogeneity and randomness, and the application of these theories to illustrative examples.

Deterministic, homogeneous, uniaxial creep rupture theory has been developed along two major approaches: I. The continuum mechanical approach, and II. The physical metallurgical approach. The first approach has been used by researchers specializing in continuum and structural mechanics, and in applied mathematics. For the most part such researchers have been interested in the macroscopic phenomenon of rupture due to creep, and have sought an interpretation of the characteristics of uniaxial creep strain on rupture data curves. The second

approach has been used by researchers specializing in metallurgy and solid state physics. They have been interested in quantitative and qualitative studies of the microscopic damage which leads to rupture. Multiaxial creep rupture theory is then usually obtained by a tensorially reasonable extension of uniaxial theory with the damage itself usually still treated as a scalar, although in some more general formulations the damage has been treated as a tensor.

In the present study we first discuss various deterministic, homogeneous, strain-independent, uniaxial and multiaxial creep rupture theories. Then the deterministic, homogeneous, uniaxial tensile, strain-dependent creep damage theory recently developed by Belloni, Bernasconi, Cozzarelli and Piatti [1,2,3], using both the continuum mechanical and physical metallurgical approaches, is discussed in detail, and generalized to the cases of compressive and multiaxial stress. The problem of the development of a failure front in a beam under pure bending is studied using this strain-dependent creep damage theory. A local damage law is then obtained for inhomogeneous materials under constant tensile stress, by means of an analysis of creep damage data for specimens of various lengths, and this law is extended to the case of variable tensile or compressive stress. This local damage law for variable stress is used to obtain solutions in a bar under constant load accompanied by lateral contraction. Finally, for the problems previously treated deterministically, the effect of randomness in the material parameters on the rupture time is analyzed by probability techniques.

Section 2 contains the literature survey of previous creep rupture theories. The strain-dependent creep damage theory and its

application to a beam under pure bending is presented in Section 3. In Section 4 we obtain the local damage law for inhomogeneous materials, and solutions are obtained for various loading conditions. Finally, the randomness of the material parameters is considered in Section 5, and an overall summary is given in Section 6.

2. LITERATURE SURVEY OF PREVIOUS CREEP RUPTURE THEORIES

As mentioned before, there are two major approaches to the development of uniaxial creep rupture theories, i.e., continuum mechanical and physical metallurgical. An early accomplishment in the continuum mechanical approach was due to Hoff [4], who assumed a ductile mode of creep rupture. This phenomenon occurs at high tensile stresses as the cross-sectional area A narrows in time due to lateral contraction and theoretically decreases to zero at rupture; such ductile rupture generally occurs in a short period of time. The logarithmic strain-displacement relation for large deformation, along with an assumption of incompressible lateral contraction for a bar under constant load, yields the relation

$$\sigma = \sigma_o e^{\varepsilon_c} \tag{1}$$

where σ is the time-dependent stress, σ_o is the initial stress, and ε_c is the time-dependent creep strain. In addition, Hoff employed the Norton's law for steady creep

$$\dot{\varepsilon}_c = K\sigma^n \tag{2}$$

where K and n are material creep parameters. Employing the ductile rupture criterion ($A \to 0$ and thus $\varepsilon_c \to \infty$ at rupture) together with Equations (1,2), Hoff obtained an expression for the ductile rupture time t_R as

$$t_R = \frac{1}{n K \sigma_o^{\,n}} \tag{3}$$

which exhibits a linear behavior when plotted as $\ln \sigma_o$ versus $\ln t_R$.

In practice, materials are frequently subjected to tensile stresses below that required for ductile rupture, and it takes a longer time for a bar to fail and it does so without obvious lateral contraction. This mode of failure is called brittle creep rupture. In order to describe this type of rupture, Kachanov [5] accounted for material deterioration (or damage) by introducing a quantity called the continuity ψ in accordance with

$$\psi = \frac{A_r}{A_o} \tag{4}$$

where A_o is the initial cross-sectional area and A_r is the effective undamaged cross-sectional area. Since the damage is assumed to be distributed over the material rather than concentrated at a crack, this approach is sometimes called a distributed damage theory (see Krajcinovic [6]). Instead of the continuity it is generally more convenient to employ the material damage parameter

$$\omega = 1 - \psi = \frac{A_o - A_r}{A_o} \tag{5}$$

and Kachancv postulated for the damage rate the power law

$$\dot{\omega} = B\left[\frac{\sigma}{1 - \omega}\right]^{\nu} \tag{6}$$

where B and ν are material damage constants. If we ignore lateral contraction under constant load, σ is simply equal to the constant stress $\sigma_o = \frac{P_o}{A_o}$ where P_o is the constant load. Equation (5) indicates

that if we assume that there is no damage in the initial state then at $t = 0$ $\omega = 0$ (or $A_r = A_o$), and also that at rupture $\omega \to 1$ (i.e., $A_r = 0$) although this theoretical limit is never actually reached. Using this brittle rupture criterion and Equation (6), the rupture time for purely brittle creep rupture at constant stress, is obtained as

$$t_R = \frac{1}{B(1+\nu)\sigma_o^{\nu}} \tag{7}$$

This relation again shows a linear behavior in a $\ln \sigma_o$ versus $\ln t_R$ plot.

Kachanov also used Equations (1) and (2) for $\sigma(t)$ in Equation (6) for the general constant load test with lateral contraction included. This is called a one-way damage creep interaction, since deformation is assumed to affect the rate of damage, while damage is assumed not to affect the rate of deformation. The rupture time is obtained from this approach as

$$t_R = t_{RH}\left\{1 - \left[1 - \left(1 - \frac{\nu}{n}\right)\frac{t_{RK}}{t_{RH}}\right]^{\frac{n}{n-\nu}}\right\} \tag{8}$$

where t_{RH} and t_{RK} are the Hoff and Kachanov rupture times given by Equations (3,7) respectively.

Following Hoff's and Kachanov's basic ideas, many researchers have introduced improvements in either the creep law and/or the damage law. Rabatnov [7] proposed the two-way damage creep interaction equations

$$\dot{\varepsilon}_c = K\,\sigma^n (1 - \omega)^{-q} \tag{9}$$

$$\dot{\omega} = B\,\sigma^{\nu} (1 - \omega)^{-r} \tag{10}$$

where q and r are material powers, and where the presence of ω in Equation (9) may be interpreted as an inclusion of tertiary creep since $\dot{\varepsilon}_c$ increase with time. From these equations, one now obtains the rupture time simply as

$$t_R = \frac{1}{B(1+r)\sigma_o^{\nu}} \tag{11}$$

and in addition the creep strain at rupture is obtained as

$$\varepsilon_R = \frac{1}{1 + r - q}\,\frac{K}{B}\,\sigma_o^{n-\nu} \tag{12}$$

Odqvist [8] added a term to the creep law to account for nonlinear instantaneous deformation (and for transient creep in an approximate manner), i.e.,

$$\dot{\varepsilon}_c = K_o \frac{d}{dt}(\sigma^{n_o}) + K\,\sigma^n \tag{13}$$

where K_o and n_o are instantaneous deformation material parameters. Broberg [9] also added a term to Rabotnov's Equation (10) to account for instantaneous damage, and proposed the modified pair of equations

$$\dot{\varepsilon}_c = G'(S)\dot{S} + F(S) \tag{14}$$

$$\dot{\omega} = g'(S)\dot{S} + f(S) \tag{15}$$

where $S = \sigma/(1 - \omega)$. Boström et al. [10] proposed a logarithmic definition of damage by analogy with the logarithmic strain as

$$\omega = \ln \frac{A_o}{A_r} \tag{16}$$

Westlund [11] and Belloni et al. [12] have given interesting comparison of the various theories developed from the Hoff and Kachanov approaches. In the next section we shall present in detail the recent strain-dependent creep damage theory of Belloni, Bernasconi, Cozzarelli and Piatti [1,2,3].

In addition to the developments following Hoff's and Kachanov's damage theories, there are also some approaches from the energy point of view. Kopecki and Walczak [13] used the dissipated energy as the rupture criterion, i.e., at rupture

$$f(E_D) = \text{constant} = \bar{K} \tag{17}$$

where E_D is the specific dissipated energy and $\bar{K}$ is a material rupture constant. It is postulated in [13] that if the energy barrier as expressed by Equation (17) has been overcome, the material loses its ability to dissipate energy and rupture occurs. Janson and Hult [14] employed a combined approach with a single defect along with distributed damage, and the strain energy was expressed in terms of an effective stress S . It is postulated in [14] that when this energy reaches a critical value equal to the work needed to create two new separate surfaces (i.e., a single crack) the material will rupture.

The physical metallurgical approach has been employed by Hult et al. [15], where an attempt was made to relate damage to the change in the velocity of sound in the material. Also, the variation of electrical resistivity was used as an index of damage. Furthermore, neutron diffraction or x-ray diffraction techniques have also been used to

measure damage. Piatti et al. [16] have developed very sensitive differential density measurement techniques as an index of damage due to void nucleation and growth, and this is discussed further in Section 3. Finally, the tedious method of actual void counting with the aid of optical and electron microscopes has also been employed.

Due to the complexity of creep rupture under multiaxial states of stress, there is at present no completely satisfactory three-dimensional theory. In fact, even the elementary one-dimensional theory is complicated by the fact that much more damage occurs under tension than under compression (e.g., see Schiller et al. [17]). Odqvist [18] generalized the uniaxial theory by replacing the uniaxial stress in the damage law by the maximum principal tensile stress in accordance with

$$\int_0^{t_R} \left|\frac{\sigma_{max}}{\sigma_K}\right|^{\nu} dt = t_K \tag{18}$$

where σ_{max} is the maximum principal tensile stress, and t_K is the rupture time under the constant uniaxial stress σ_K. However, Johnson, Henderson and Khan [19] and Henderson and Mathur [20,21] have experimentally demonstrated that the rupture time of different materials appears to depend on different rupture criteria. Hayhurst [22] has shown that copper and aluminium alloys demonstrate two extreme types of rupture behavior under multiaxial stress. For copper a maximum tensile stress criterion such as Equation (18) was found to be valid, while for aluminium alloys a maximum shear stress criterion was more appropriate.

Martin and Leckie [23] and Hayhurst and Leckie [24] have proposed a combined theory which included these two limiting cases. For homogeneous damage (i.e., the effect of damage is included in all three principal stresses) a potential function ϕ was assumed as

$$\phi = \left(\frac{\dot{\varepsilon}_o}{\sigma_o^{\,n}}\right)^{\frac{1}{n+1}} \left(\frac{\sigma_I - \sigma_{III}}{\psi}\right) \tag{19}$$

The principal strain rates were derived from ϕ as

$$\dot{\varepsilon}_I = \dot{\varepsilon}_{III} = \dot{\varepsilon}_o \left(\frac{\sigma_I - \sigma_{III}}{\psi\,\sigma_o}\right)^n \tag{20a}$$

$$\dot{\varepsilon}_{II} = 0 \tag{20b}$$

where σ_I , σ_{II} and $\sigma_{III}(\sigma_I > \sigma_{II} > \sigma_{III})$ are the principal stresses, $\dot{\varepsilon}_I$, $\dot{\varepsilon}_{II}$ and $\dot{\varepsilon}_{III}$ are the corresponding principal strain rates, and $\dot{\varepsilon}_o$, σ_o are nominal values. Then the damage law was given by

$$\dot{\psi} = -B\left(\frac{\sigma_I - \alpha\,\sigma_{III}}{\psi\,\sigma_o}\right)^{\nu} \tag{21}$$

and it follows that for $\alpha = 0$ Equation (21) corresponds to a maximum tensile stress criterion and for $\alpha = 1$ Equation (21) corresponds to a maximum shear stress criterion. For nonhomogeneous damage (i.e., the effect of damage is included only in σ_I) the potential function was given by

$$\phi = \left(\frac{\dot{\varepsilon}_o}{\sigma_o^{\,n}}\right)^{\frac{1}{n+1}} \left(\frac{\sigma_I}{\psi} - \sigma_{III}\right) \tag{22}$$

The principal strain rates then were derived from Equation (22) as

$$\dot{\varepsilon}_{I} = -\dot{\varepsilon}_{III} = \dot{\varepsilon}_{o}\left[\left(\frac{\sigma_{I}}{\psi} - \sigma_{III}\right)/\sigma_{o}\right]^{n} \tag{23a}$$

$$\dot{\varepsilon}_{II} = 0 \tag{23b}$$

and the damage law was

$$\dot{\psi} = -B\left[\left(\frac{\sigma_{I}}{\psi} - \theta\ \sigma_{III}\right)/\sigma_{o}\right]^{\nu} \tag{24}$$

The previous multiaxial theories express the material damage in terms of scalar quantities. On the other hand, Murakami and Ohmo [25] have assumed that material damage can be expressed as a symmetric tensor of rank two. Recently, a tensor of rank eight was proposed by Chaboche [26] for the multiaxial material damage. This damage tensor was reduced to a tensor of rank two using the symmetry of the stress tensor. In the following section we present a recent damage theory, in which strain plays a significant role.

3. THE STRAIN-DEPENDENT CREEP DAMAGE THEORY

In the previous section all of the damage laws were expressed as independent of the strain, but recent experimental results obtained by Belloni, Bernasconi and Piatti at the Euraton Laboratory [1] suggest that strain may in fact be a dominant variable in damage accumulation. These results establish the connection between relative density variation, which is taken as a measure of damage, and the creep strain, temperature, stress and time. Thus, the damage law is in general expressed as

$$\frac{-\Delta\rho}{\rho_o} = f(\epsilon_c, T, \sigma, t) \tag{25}$$

where ρ_o is the initial density, $-\Delta\rho$ is the reduction in density due to void formation and growth, and T is the temperature.

3.a Strain-Dependent Creep Damage Power Law

The strain-dependent creep damage theory presented in [1] has been experimentally verified using very sensitive differential density measurement techniques [27-29]. By these techniques the damaged specimen and an undamaged dummy ar[illegible]ed both in air and in a heavy liquid using an analytical balance, and then relative density variations as small as 10^{-5} or even 10^{-6} are computed from a simple formula. By using an extensive computer statistical regression analysis, a good agreement was established with the experimental results for the case of one dimensional constant tensile stress (as approximated by constant load), using the separable power law form for the damage law

$$-\frac{\Delta\rho}{\rho_o} = C_o\, \varepsilon_c^{\alpha}\, e^{-\beta/T}\, \sigma_o^{\gamma} t^{\delta} \tag{26}$$

Equation (26) simplifies to

$$-\frac{\Delta\rho}{\rho_o} = C\, \varepsilon_c^{\alpha}\, \sigma_o^{\gamma} t^{\delta} \tag{27}$$

if the temperature is constant at T_o and C is the constant $C = C_o e^{-\beta/T_o}$. In Equations (26,27) α, β, γ, δ are exponents which characterize respectively the effects of creep strain, temperature, stress and time on the damage. The experimental data indicate that the creep strain is always the most important factor among these four damage variables. Also, the four exponents were shown to be positive constants, which depend on the material and, within certain limits, do not depend on the test conditions. Also, in [1] the Kachanov damage parameter ω was shown to be essentially equal to $-\frac{\Delta\rho}{\rho_o}$ by means of a simple conservation of mass calculation.

In the present study, Equation (27) will be rewritten in the more general form

$$D(t) = C\, \varepsilon_c^{\alpha}\, \sigma_o^{\gamma} t^{\delta} \tag{28}$$

where $D(t)$ denotes the damage and does not necessarily correspond with $-\frac{\Delta\rho}{\rho_o}$. Combining Equations (2,28) (i.e., excluding transient creep) one obtains the rupture time for constant tensile stress as

$$t_R = \left(\frac{D_R}{C\, K^{\alpha} \sigma_o^{n\alpha+\gamma}}\right)^{\frac{1}{\alpha+\delta}} \tag{29}$$

where D_R is the critical value of damage at rupture. This is again a linear relation in a $\ln \sigma_o$ versus $\ln t_R$ plot. By assuming that

the rupture times in Equations (7,29) are the same, one can establish a correspondence between Kachanov's theory and Equation (28), yielding

$$\nu = \frac{n\alpha+\gamma}{\alpha+\delta} \tag{30}$$

$$B = \frac{1}{1+\nu}\left[\frac{C\ K^{\alpha}}{D_R}\right]^{\frac{1}{\alpha+\delta}} \tag{31}$$

This is an important result since it makes it possible to interpret Kachanov's theory from the metallurgical point of view.

Cozzarelli and Bernasconi [2] extended Equation (28) to the case of variable tensile stress $\sigma(t)$ as

$$D(t) = C\ \varepsilon_c(t)^{\alpha}\left[\int_0^t \sigma(t')^{\gamma/\delta}dt'\right]^{\delta} \tag{32}$$

where for $\sigma(t) = \sigma_0$ Equation (32) reverts to the form in Equation (28). Furthermore, it possesses the physically necessary characteristic that D(t) be continuous even if the stress history is discontinuous. Using Equation (32), rupture times were also determined for various loading conditions, such as step-stress, constant stress rate, relaxation and constant load with lateral contraction. Hult [30] and Chrzanowski [31] assumed that there is no damage produced under compression, and in [17] it was shown that the experimental data essentially justify such an assumption. Hence, Equation (32) can be generalized further as

$$D(t) = C\ \varepsilon_c(t)^{\alpha}\left\{\int_0^t \sigma(t')^{\gamma/\delta}U[\sigma(t')]dt'\right\}^{\delta} \tag{33}$$

where $\sigma(t)$ can be either tensile or compressive, and $U(\sigma)$ is the unit step function.

Although multiaxial strain-dependent creep damage is not of major interest in the present study, we shall propose a multiaxial law for possible future study. Since rupture will occur when the maximum damage reaches a critical value, the material damage will simply be expressed in terms of scalar quantities which will produce this maximum damage. We assume that a one-way damage creep interaction is valid, that the principal axes remain fixed during the creep process, and that either the maximum tensile stress or the maximum shear stress criterion is valid. The damage law (28) is then generalized for multiaxial states of stress as

$$D = C\,\varepsilon_c^{*\alpha}\sigma^{*\gamma}t^{\delta} \tag{34}$$

where, for a maximum tensile stress criterion, ε_c^* and σ^* are the maximum principal strain and stress and are given by (see Finnie and Heller [32])

$$\varepsilon_c^* = BJ'^{\,n-1}[\sigma_I - \tfrac{1}{2}(\sigma_{II} + \sigma_{III})] \tag{35a}$$

$$\sigma^* = \sigma_I \tag{35b}$$

where $J' = \frac{1}{\sqrt{2}}\left[(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2\right]^{1/2}$. On the other hand, for a maximum shear criterion, ε_c^* and σ^* are the maximum shear strain and stress, and are given by

$$\varepsilon_c^* = \frac{3}{2}\,BJ'^{\,n-1}(\sigma_I - \sigma_{III}) \tag{36a}$$

$$\sigma^* = \frac{1}{2}(\sigma_I - \sigma_{III}) \tag{36b}$$

In Section 3.b, we use Equation (33) to study the case of a beam under pure bending.

3.b Damage in a Beam under Pure Bending

Consider an initially straight beam, with a uniform rectangular cross-section of width w and initial effective depth $2h_o$, and subjected to pure bending by constant moment M. As mentioned in the last section, there is essentially no damage produced under compression and therefore the beam will deteriorate only on that side of the neutral axis that is under tensile stress. Since the maximum tensile stress occurs at the top outside fiber (see Figure 1), rupture will initiate at this fiber and then a failure front (see [5]) will propagate down through the entire cross-section. The neutral axis is at the centroid of the undamaged material, and we denote the initial neutral axis as x_o and the initial axis in the direction of the depth as y_o. As rupture develops and the failure front propagates the neutral axis moves down and is now identified as x; the intersection of x with y_o defines the moving origin of the instantaneous depth coordinate y.

In order for Norton's law (2) to be valid for both tensile and compressive stresses, it is rewritten in the form

$$\dot{\varepsilon}_c = K|\sigma|^n \operatorname{sgn}(\sigma) \tag{37}$$

where $\operatorname{sgn}(\sigma)$ is the signum function defined as

$$\operatorname{sgn}(\sigma) = \begin{cases} 1 & \text{for } \sigma > 0 \\ -1 & \text{for } \sigma < 0 \end{cases} \tag{38}$$

and where n is not necessarily restricted to odd positive integers. Employing the Euler-Bernoulli assumption of plane sections remaining plane, we obtain the strain rate $\dot{\varepsilon}_c$ in terms of the curvature rate $\dot{\kappa}$ as

$$\dot{\varepsilon}_c = y\dot{\kappa} \tag{39}$$

An expression for the bending moment is then found from Equations (37,39) as

$$M = \int \sigma y dA = \left|\frac{\dot{\kappa}}{K}\right|^{\frac{1}{n}} J \operatorname{sgn}(\dot{\kappa}) \tag{40}$$

where

$$J = \int |y|^{1+\frac{1}{n}} dA \tag{41}$$

which depends on the geometry of the cross-section and the creep power n . For a beam of rectangular cross-section with width w and *instantaneous* depth $2h$, we have

$$J = \frac{2n}{2n+1} wh^{2+\frac{1}{n}} \tag{42}$$

For $n = 1$ this reduces to $\frac{2}{3}wh^3$, which is the ordinary second moment of area for a rectangular beam. The stress is obtained from a combination of Equations (37,39,40) as

$$\sigma = \frac{M|y|^{\frac{1}{n}}}{J} \operatorname{sgn}(y) \tag{43}$$

Initially, the maximum tensile stress σ_{max} in the beam occurs at the top outside fiber $y = y_o = h_o$, and is given by

$$\sigma_{max} = \frac{M(h_o)^{\frac{1}{n}}}{J_o} \tag{44}$$

where $J_o = \frac{2n}{2n+1} w h_o^{2+\frac{1}{n}}$.

The entire cross-section of the beam will resist the applied bending moment until a failure front forms at $y = h_o$. Since the maximum tensile stress will remain constant up to the time of initial rupture, this time is given by Equation (29) as

$$t_{RI} = \left\{ \frac{D_R}{C\ K^{\alpha}} \left[\frac{M(h_o)}{J_o} \right]^{\frac{1}{n}(-(n\alpha+\gamma))} \right\}^{\frac{1}{\alpha+\delta}} \tag{45}$$

Beginning at $t = t_{RI}$ the failure front will travel downward, and the maximum tensile stress is then at the failure front and travels along with it. During this rupture propagation process the maximum tensile stress is no longer a constant. Figure 1 shows that at any time $t > t_{RI}$ the thickness of the deteriorated region is $2\zeta(t)$, and the *effective depth of the beam has been reduced to a value* $2h(t) < 2h_o$. It is clear from the figure that $h_o = h(t) + \zeta(t)$ and $y(t) = y_o(t) + h_o - h(t)$, where $y(t)$ is measured from the shifted position of the neutral axis. The distribution of stress is then given by

$$\sigma(t) = \frac{M|y(t)|^{\frac{1}{n}}}{J(t)} \ \mathrm{sgn}(y) \tag{46}$$

and the strain is

$$\epsilon_c(t) = K \int_0^t |\sigma(t')|^n \,\mathrm{sgn}(\sigma)\,dt' \tag{47}$$

From Equations (33,46,47), we finally obtain the damage at $y(t) > 0$ as

$$D(t) = C\left\{ KM^n \int_0^t \frac{[y_o(t)+h_o-h(t')]}{J(t')^n}\,dt' \right\}\left\{ M^{\gamma/\delta} \int_0^t \frac{[y_o(t)+h_o-h(t')]^{\gamma/\delta n}}{J(t')^{\gamma/\delta}}\,dt' \right\}^{\delta} \tag{48}$$

Also, at the time $t > t_{RI}$, with the failure front at the position $y = h(t)$, it is clear that at this front $y_o(t) + h_o = 2h(t)$ and $D(t) = D_R$. Therefore, Equation (48) yields

$$D_R = C\,K^{\alpha}\,M^{n\alpha+\gamma}\left\{\left(\frac{2n+1}{2nw}\right)^{n}\int_o^t [2h(t)-h(t')h(t')^{-1-2n}\,dt'\right\}^{\alpha}$$
$$\times\left\{\left(\frac{2n+1}{2nw}\right)^{\gamma/\delta}\int_o^t [2h(t)-h(t')]h(t')^{-\frac{\gamma}{\delta n}-\frac{2\gamma}{\delta}}\,dt'\right\}^{\delta} \qquad (49)$$

From the experimental data given in [3], we find that although in general $\gamma \gtrless \delta n$ the value of γ is in the vicinity of δn. For the sake of simplicity we shall assume here that $\gamma = \delta n$ (also see [2]), in which case Equation (49) then simplifies to

$$D_R = C\,K^{\alpha}\,M^{n\alpha+\gamma}\left\{\left(\frac{2n+1}{2nw}\right)^{n}\int_o^t [2h(t)-h(t')]h(t')^{-1-2n}\,dt'\right\}^{\alpha+\delta} \qquad (50)$$

Differentiating Equation (50) with respect to t, we find that

$$2\dot{h}\int_o^t h(t')^{-1-2n}\,dt' + h^{-2n} = 0 \qquad (51)$$

where $\dot{h}$ designates dh/dt. In addition, we have the initial condition

$$h = h_o \quad \text{at} \quad t = t_{RI} \qquad (52)$$

Upon substituting Equation (52) back into Equation (51), another initial condition is found as

$$\dot{h} = -\frac{h_o}{2t_{RI}} \quad \text{at} \quad t = t_{RI} \qquad (53)$$

A second order differential equation for h may be obtained by differentiating Equation (51) with respect to t and eliminating the integral, yielding

$$\ddot{h} + 2(n-1)\ h^{-1}(\dot{h})^2 = 0 \tag{54}$$

This equation is of the same form as that derived by Kachanov [5] and Odqvist [18]. Solving differential Equation (54) with initial conditions (52,53), we then obtain the useful result

$$\frac{t}{t_{RI}} = 1 + \frac{2}{2n-1}\left[1 - \left(\frac{h}{h_o}\right)^{2n-1}\right] \tag{55}$$

At the instant of complete rupture, i.e., when $h = 0$, the rupture time is given simply as

$$\frac{t_R}{t_{RI}} = 1 + \frac{2}{2n-1} \tag{56}$$

At the instant that the cross-section has deteriorated to half of its initial effective depth, i.e., $h = \frac{1}{2}h_o$, we obtain the half rupture time $t_{R'}$ as

$$\frac{t_{R'}}{t_{RI}} = 1 + \frac{2}{2n-1}\left[1-(2)^{-2n+1}\right] \tag{57}$$

The ratio $\frac{t_{R'}}{t_{RI}}$ as computed from Equation (57) is only slightly different from $\frac{t_R}{t_{RI}}$ [Equation (56)], and this difference decreases as n increases. This interesting result indicates that it will take only a small part of the total lifetime for the cross-section of the beam to go from a state of 50% deterioration to complete rupture, and the rupture strength of the beam decreases as n increases. Figure 2

shows $\frac{h}{h_o}$ plotted as a function of $\frac{t}{t_{RI}}$ for the following two steels [3]:

AISI 310 stainless steel at 600°C, where $n = 7$

2.25Cr 1Mo ferritic steel at 550°C, where $n = 5.6$

The figure clearly shows the acceleration of the propagation of the failure front and the weakening of the beam as n increases. As indicated in [18], when half the depth of the beam has deteriorated, the rupture process becomes more and more ductile due to the development of a high stress distribution. Since the above development does not contain a Poisson effect such as in the ductile rupture theory of Hoff, it is less accurate in this latter time interval.

In the next section, the inhomogeneity of the material will be considered in order to further improve the strain-dependent creep damage theory.

4. INHOMOGENEOUS STRAIN-DEPENDENT CREEP RUPTURE

The theories in the previous two sections all assume that the material is homogeneous, and accordingly the material parameters are independent of position. However, in actual materials there are numerous imperfections, which results in significant inhomogeneity in the structure sensitive creep parameters. It was noted by Broberg [33] that both the Hoff and Kachanov theories, as applied to a bar in tension, have a tendency to overestimate the lifetime to rupture. He showed that this may be attributed to variations of the creep strain and creep damage parameters (K and B) along the axis of the bar resulting from inhomogeneity in the material. Similarly, we shall clearly demonstrate in this section that in the strain-dependent creep damage theory the inhomogeneity of the material (coupled with the fact that the experimental values of the critical damage D_R are measured over a finite distance rather than at a point) can account for the fact that D_R is far less than the theoretical limit value of 1.

4.a Inhomogeneous Strain-Dependent Damage Law for Constant Stress

The problem of spatial fluctuations (generally random) in the material creep parameters is discussed in detail in Cozzarelli [34], where it is noted that for simplicity one may in some cases neglect spatial fluctuation in the creep power n while allowing only the creep reciprocal viscosity K to fluctuate (as in Broberg [33] and Broberg and Westlund [35]). Extending this concept to the present

creep rupture problem, we assume that all of the powers n, α, γ, δ are constant while K and the damage coefficient C may fluctuate spatially due to material inhomogeneity. For one dimensional constant tensile stress, the Norton creep law then takes the form

$$\varepsilon_c(x,t) = K(x)\sigma_o{}^n t \tag{58}$$

where $K(x)$ depends on the axial coordinate x. In order to account for inhomogeneous damage that is present prior to loading, we introduce the concept of a damage difference $\tilde{D}$, which is defined as

$$\tilde{D}(x,t) = D_T(x,t) - D_o(x) \tag{59}$$

where $D_T(x,t)$ is the total damage and $D_o(x)$ is the initial damage at $t = 0$. At a particular constant temperature T_o, we postulate by extension of strain-dependent law (28) the local damage law

$$\tilde{D}(x,t) = C(x)\varepsilon_c(x,t)^{\alpha}\sigma_o{}^{\gamma} t^{\delta} \tag{60}$$

Upon substituting Equation (58) into Equation (60), we then obtain

$$\tilde{D}(x,t) = H(x)\sigma_o{}^{n\alpha+\gamma} t^{\alpha+\delta} \tag{61}$$

where we have introduced the combined material parameter $H(x) = C(x)K(x)^{\alpha}$.

For a particular applied constant tensile stress σ_c at constant temperature T_o, a one-dimensional specimen of length ℓ will reach rupture at some position x_o and at a particular time t_{CR}. In the neighborhood of this position we may compute the average damage at rupture $\tilde{D}_R^{AV}(x)$ over interval $x_o \leq x' \leq x$ as

$$\tilde{D}_R^{AV}(x) = \frac{\int_{x_o}^{x} \tilde{D}_R(x', t_{CR})\,dx'}{x - x_o} = \frac{\int_{x_o}^{x} H(x')\,dx'}{x - x_o} Q \qquad (62)$$

where $Q = \sigma_C^{n\alpha+\gamma} t_{CR}^{\alpha+\delta}$. The experimental results given in [17] contain very useful data for the fractional density change plotted as a function of the mass for various one dimensional specimens of different lengths cut from the test specimen at the position of rupture. These data thus provide the variation of $\tilde{D}_R^{AV}(x)$ with ρAx ; for simplicity we will at present place x_o at the origin (i.e., $x_o = 0$) and also assume that the initial damage $D_o(x) = 0$. We found that $\tilde{D}_R^{AV}$ can be approximated very nicely by means of a least squares curve fit as the simple expression

$$\tilde{D}_R^{AV}(x) = a + \frac{b'(1-e^{-c'\rho Ax})}{\rho Ax} \qquad (63)$$

where a, b', c' are positive constants for a particular material. As an example we used the constant stress data in [17] for AISI 310 stainless steel at temperature $T_o = 600°C$ and constant strain rate $\dot{\varepsilon}_c = 27.8 \times 10^{-6}$/sec, and obtained the values $a = 0.0015662$, $b' = 0.0013289$, and $c' = 33.048296$ with $\rho = 7.8$ Kg/mm^3 and $A = 12.6 \times 10^2$ mm^2. From Equations (62,63) we can now obtain for the combined material parameter the simple exponential expression

$$H(x) = \frac{1}{Q}(a + b\, e^{-cx}) \qquad (64)$$

where $b = b'c'$ and $c = c'\rho A$ in general, and $b = 4.3918 \times 10^{-2}$ and $c = 3.2480 \times 10^5$ in the above example.

For the above data, Figure 3 shows a good agreement between $\tilde{D}_R^{AV}(x)$ as calculated from Equation (63) and the actual data points from Ref. [17]. The figure also contains a plot of $H(x)\times Q$ in accordance with Equation (64). From this curve [and Equation (64)] we see that $H(x)\times Q$ reaches a peak of magnitude b superposed on top of the constant value a. We may thus interpret the quantity b as a measure of the strength of a local imperfection. The local inhomogeneous damage law is now given by a combination of Equations (61) and (64), yielding

$$\tilde{D}(x,t) = \frac{1}{Q}(a + be^{-cx})\sigma_o^{n\alpha+\gamma}t^{\alpha+\delta} \qquad (65)$$

It follows from Equation (63) and (65) that under the constant tensile stress σ_c the local critical damage is

$$\tilde{D}_R(0,t_{CR}) = \tilde{D}_R^{AV}(0) = a + b \gtrsim \tilde{D}_R^{AV}(x) \qquad (66)$$

where $x = x_o = 0$ corresponds to the rupture point. It is clear from Equation (66) that the critical value of damage at the rupture point is always greater than or equal to the average value. We also have for the asymptotic value of $\tilde{D}_R^{AV}$

$$\tilde{D}_R^{AV}(\infty) = D_R = a \qquad (67)$$

which is in fact the experimental value of the critical damage reported in [1]. From the values of a and b presented above for AISI 310 stainless steel at 600°C and Equations (66,67), we obtain for the local critical value of damage $\tilde{D}_R = a + b \approx 4.5 \times 10^{-2}$ and for the experimental critical value of damage $D_R = a \approx 1.5 \times 10^{-3}$. Clearly, $\tilde{D}_R$ is significantly closer to the theoretical limiting value of 1 than

is $\tilde{D}_R$. This value of $\tilde{D}_R$ could be improved further if we had additional experimental data in the smaller interval $0 \leq \rho\Delta x \leq 0.1$.

If we do not place x_o at the origin, we can simply modify Equation (65) for $x_o \leq x \leq \ell$ as follows

$$\tilde{D}(x,t) = \frac{1}{Q}[a + be^{-c(x-x_o)}]\sigma_o^{n\alpha+\gamma}t^{\alpha+\delta} \tag{68}$$

Since Equation (68) is valid only on one side of x_o , we may extend it further to the case $x \gtrless x_o$ by assuming that the damage is symmetrically distributed about x_o . We thus obtain for $0 \leq x \leq \ell$

$$\tilde{D}(x,t) = H(x)\sigma_o^{n\alpha+\gamma}t^{\alpha+\delta} \tag{69a}$$

with

$$H(x) = \frac{1}{Q}(a + be^{-c|x-x_o|}) \tag{69b}$$

For each value of x there is a corresponding rupture time, and actual rupture will occur at the minimum of these rupture times. The rupture time for any applied constant tensile stress σ_o with $D_o(x)$ assumed as zero may now be obtained from Equations (69) as

$$t_R = \left[\frac{\tilde{D}_R Q}{a+b}\sigma_o^{-(n\alpha+\gamma)}\right]^{\frac{1}{\alpha+\delta}} \tag{70}$$

As mentioned in Section 3(, compression will produce essentially no damage and we can generalize Equation (65) for both tensile and compressive stresses to the form

$$\tilde{D}(x,t) = \frac{1}{Q}(a + be^{-c|x-x_o|})\sigma_o^{n\alpha+\gamma}\, t^{\alpha+\delta}\, U(\sigma_o) \tag{71}$$

If additional simplicity is required, one can approximate Equation (71) by

$$\tilde{D}(x,t) = \frac{1}{Q}\left\{a+b\left[U\left(x-x_o+\frac{d}{2}\right)-U\left(x-x_o-\frac{d}{2}\right)\right]\right\}\sigma_o^{\,n\alpha+\delta}\,t^{\alpha+\delta}\,U(\sigma_o) \tag{72}$$

which distributes the damage uniformly over a small region with center at x_o and of width d . In this case one obtains the same rupture time as in Equation (70), but rupture will now occur at any point within this region.

We shall now extend the constant load solution for homogeneous materials presented in [2], using the above inhomogeneous formulation.

4.b Inhomogeneous Creep Rupture Under Constant Load-Large Deformation

If lateral contraction due to the Poisson effect is not ignored, the cross-sectional area of a bar under a constant tensile load P_o is clearly a decreasing function of time. As a result, the stress increases with time in accordance with

$$\sigma(x,t) = \frac{P_o}{A(x,t)} \tag{73}$$

where the area $A(x,t)$ may in general also vary with the axial coordinate x . In order to generalize inhomogeneous damage Equation (60) to the case of variable stress, we first rewrite it in the form

$$\frac{\partial}{\partial t}\left[\frac{\tilde{D}(x,t)}{C(x)\varepsilon_c(x,t)^{\alpha}}\right]^{\frac{1}{\delta}} = \sigma_o^{\gamma/\delta} \tag{74}$$

Then by analogy with the procedure employed for homogeneous damage (see [2]), we postulate that for variable stress the σ_o in Equation (74) may be simply replaced by $\sigma(x,t)$. Integrating this result with the initial condition $\tilde{D}(x,0) = 0$, we obtain the integral form

$$\tilde{D}(x,t) = C(x)\,\epsilon_c(x,t)^{\alpha}\left[\int_0^t \sigma(x,t')^{\gamma/\delta}dt'\right]^{\delta} \tag{75}$$

Equation (75) is consistant with the postulate that the rate of damage be a function of the creep strain rate, creep strain, damage and stress, i.e.

$$\frac{\partial}{\partial t}\tilde{D}(x,t) = f\left[\frac{\partial}{\partial t}\epsilon_c(x,t),\ \epsilon_c(x,t),\ \tilde{D}(x,t),\ \sigma(x,t)\right] \tag{76}$$

For $\sigma(x,t) = \sigma_0$ Equation (75) clearly reduces to Equation (60).

The total lateral strain is given as in Courtine et al. [36] by

$$\epsilon_{\ell}(x,t) = -\left[\nu_e\frac{\sigma(x,t)}{E} + \nu_s\,\epsilon_c(x,t)\right] \tag{77}$$

where E is the elastic modulus, and ν_e and ν_s are the elastic and steady creep Possion's ratios respectively. For simplicity we shall neglect transient creep, but this can be corrected in an approximate manner by replacing E with E^*, a smaller fictitious modulus of elasticity (see [2]). The creep strain for an inhomogeneous material then follows from Equation (58) simply as

$$\epsilon_c(x,t) = K(x)\int_0^t \sigma(x,t')^n\,dt' \tag{78}$$

By a combination of Equations (77,78) we obtain the result

$$\frac{\partial}{\partial t}\epsilon_{\ell}(x,t) = -\frac{\nu_e}{E}\frac{\partial}{\partial t}\left[\frac{P_0}{A(x,t)}\right] - \nu_s K(x)\left[\frac{P_0}{A(x,t)}\right]^n, \quad t > 0 \tag{79}$$

For the strain rate we use the logarithmic form for finite deformation

$$\frac{\partial}{\partial t}\epsilon_{\ell}(x,t) = \frac{\partial R(x,t)/\partial t}{R(x,t)} = \frac{1}{2}\frac{\partial A(x,t)/\partial t}{A(x,t)} \tag{80}$$

where $R(x,t)$ is the instantaneous cross-sectional radius. This leads to the partial differential equation in $A(x,t)$

$$\frac{1}{2}\,\frac{\partial A/\partial t}{A} = \frac{\nu_e P_o}{E}\,\frac{\partial A/\partial t}{A} - \nu_s K(x) P_o{}^n \left(\frac{1}{A}\right)^n \tag{81}$$

which at some constant value of $x = x_c$ may be rewritten in the ordinary differential form

$$\nu_s K P_o{}^n dt = \frac{\nu_e P_o}{E} A^{n-2}\, dA - \frac{1}{2} A^{n-1}\, dA, \quad t > 0 \ , \quad x = x_c \tag{82}$$

Integrating Equation (82) with initial condition $A(x,0^+) = A_o$, we obtain the solution

$$\left(\frac{A}{A_o}\right)^n \left[1 - \frac{2n\nu_e\sigma_o}{(n-1)E}\left(\frac{A}{A_o}\right)^{-1}\right] = 1 - \frac{2n\nu_e\sigma_o}{(n-1)E} - 2n\ \nu_s\ K\ \sigma_o{}^n t, \quad t > 0 \tag{83}$$

which being valid for any x_c is in fact valid for all x . Equation (83) is a solution for $A(x,t)$ in implicit form, and thus must in general be solved by numerical techniques. However, we may approximate $\frac{A}{A_o}$ within the brackets by unity, and obtain the approximate closed form solution

$$\frac{A(x,t)}{A_o} \approx \left[1 - \frac{2n(n-1)E\nu_s K(x)}{(n-1)E-2n\nu_e\sigma_o}\,\sigma_o{}^n\, t\right]^{-\frac{1}{n}} , \quad t > 0 \tag{84}$$

The stress then follows from Equations (73) and (84) as

$$\sigma(x,t) = \sigma_o\,\frac{A_o}{A(x,t)} \approx \sigma_o \left[1 - \frac{2n(n-1)E\nu_s K(x)}{(n-1)E-2n\nu_e\sigma_o}\,\sigma_o{}^n\, t\right]^{-\frac{1}{n}} \tag{85}$$

Substituting Equation (85) into Equation (78) and evaluating the integral, we also obtain the creep strain as

$$\varepsilon_c(x,t) = -\frac{\ln[1-IK(x)\sigma_o^n t]}{[illegible]} \qquad (86)$$

where $I = \frac{2n(n-1)E_s}{(n-1)n-2n\varepsilon_c\sigma_o}$. At $t = \frac{1}{IK(x)\sigma_o^n}$ $\varepsilon_c \to \infty$, which is similar to Hoff's ductile rupture criterion. The damage may now be obtained from Equations (75,85) as

$$D(x,t) \cong C(x)\varepsilon_c(x,t)^\alpha \sigma_o^\gamma \frac{\delta n}{(\gamma-\delta n)} \frac{1}{IK(x)\sigma_o^n}\left[(1-IK(x)\sigma_o^n t)^{\frac{\delta n-\gamma}{\delta n}}-1\right]^\delta \qquad (87)$$

For the special case $\gamma = \delta n$ (see discussion in Section 3), Equation (87) simplifies to

$$\tilde{D}(x,t) \cong C(x)\varepsilon_c(x,t)^\alpha \sigma_o^\gamma \left[-\frac{1}{IK(x)\sigma_o^n}\ln(1-IK(x)\sigma_o^n t)\right]^\delta \qquad (88)$$

At each position x and corresponding to a critical value of rupture D_R there is a rupture time $t_R(x)$ expressed by Equation (87), which is an implicit function of x and $t_R(x)$. Actual rupture will occur at that position in the bar where there is a minimum value of $t_R(x)$. Thus, in general the location and time of rupture is determined by a minimization of $t_R(x)$ in Equation (87). In the special homogeneous case $D_o(x) = 0$ and $K(x) = K$, for which Equation (69b) is valid, we know a priori that rupture will occur at $x = x_o$, and that the rupture time t_R is the root of the equation

$$\tilde{D}_R = \frac{a+b}{QK^\alpha}\varepsilon_c(t_R)^\alpha \sigma_o^\gamma \left\{\frac{\delta n}{\gamma-\delta n}\frac{1}{IK\sigma_o^n}\left[(1-IK\sigma_o^n t_R)^{\frac{\delta n-\gamma}{\delta n}}-1\right]\right\}^\delta \qquad (89)$$

We shall consider in the next section inhomogeneous rupture under constant load with small deformation.

4.c Inhomogeneous Rupture Under Constant Load-Small Deformation

Let us define the area increment

$$\tilde{A}(x,t) = A_o - A(x,t) \quad , \quad \tilde{A}(x,t) << A_o \tag{90}$$

where in accordance with small deformation theory $\tilde{A}(x,t)$ is a small quantity. With transient creep still excluded, substituting Equation (90) into Equation (81) and neglecting higher order terms we obtain the linear partial differential equation with variable coefficients.

$$\left(\frac{\nu_e \sigma_o}{E} - \frac{1}{2}\right)\frac{\partial\tilde{A}/\partial t}{A_o} + n\ \nu_s K(x)\sigma_o{}^n \left(\frac{\tilde{A}}{A_o}\right) + \nu_s\ K(x)\sigma_o{}^n = 0,\ t > 0 \tag{91}$$

For convenience we rewrite Equation (91) as

$$a_1 \frac{\partial\bar{A}}{\partial t} + a_2(x)\bar{A} + \frac{a_2(x)}{n} = 0 \ , \qquad t > 0 \tag{92}$$

where $\bar{A} = \dfrac{\tilde{A}}{A_o}$, $a_1 = \dfrac{\nu_e \sigma_o}{E} - \dfrac{1}{2}$, and $a_2(x) = n\ \nu_s K(x)\sigma_o{}^n$. At some fixed position $x = x_c$, Equation (92) is the ordinary differential equation with constant coefficients

$$a_1 \frac{\partial\bar{A}}{\partial t} + a_2\ \bar{A} + \frac{a_2}{n} = 0 \ , \qquad t > 0 \ , \qquad x = x_c \tag{93}$$

Employing the initial condition $\bar{A}(x,0^+) = 0$ the solution is easily obtained as

$$\bar{A}(x,t) = e^{-\frac{a_2}{a_1}t} - \frac{1}{n} \ , \qquad t > 0 \tag{94}$$

which is valid for all x since the result is independent of the choice of x_c .

Substituting Equation (94) into Equation (73) and neglecting the higher order terms, we obtain for the stress

$$\sigma(x,t) = \sigma_o\left[1 + e^{-\frac{a_2(x)}{a_1}t} - \frac{1}{n}\right] \tag{95}$$

The expression for the creep strain then follows from Equations (75,95) as

$$\varepsilon_c(x,t) = K(x)\sigma_o^{\,n}\,\frac{na_1}{a_2(x)}\left[1 - e^{-\frac{a_2(x)}{a_1}t}\right] \tag{96}$$

Finally, the damage for small deformation is obtained from Equations (75,95) as

$$\tilde{D}(x,t) = C(x)\varepsilon_c(x,t)^{\alpha}\,\sigma_o^{\gamma}\left\{\left(1 - \frac{1}{\delta n}\right)t + \frac{\gamma a_2(x)}{\delta a_1}\left[1 - e^{-\frac{a_2(x)}{a_1}t}\right]\right\}^{\delta} \tag{97}$$

The method discussed in Section 4(b) for determining the rupture time and the location of rupture clearly also applies here. In general, for a given $\tilde{D}_R$ we must minimize $t_R(x)$ as prescribed by Equation (97), in order to locate the position where rupture will take place. Then by substituting this particular value of x back into Equation (97) we may find the value of t_R. For the special homogeneous case $D_o(x) = 0$ and $K(x) = K$, rupture will clearly occur at $x = x_o$ and the rupture time t_R is obtained from

$$\tilde{D}_R = \frac{a+b}{QK^{\alpha}}\,\varepsilon_c(t_R)^{\alpha}\,\sigma_o^{\gamma}\left[\left(1 - \frac{1}{n}t_R\right) + \frac{\gamma a_2}{a_1}\left(1 - e^{-\frac{a_2}{a_1}t_R}\right)\right]^{\delta} \tag{98}$$

In the next section we examine the effect of randomness in the material parameters on the rupture time.

5. EFFECT OF RANDOM PARAMETERS ON CREEP RUPTURE

Creep parameters are very highly dependent on the microscopic imperfection variations introduced in the manufacturing process and on the small temperature variations encountered in practice, and such variations are very difficult to control. It thus follows that this lack of precise information concerning imperfection and temperature fluctuations will be reflected in considerable uncertainty in predicting lifetime to rupture. Classical probability techniques were employed by Cozzarelli and Huang [37-39] in the study of steady creep of structures with random material parameters. Huang and Valentin [40] extended this work to the study of creep deformation and creep rupture in cylindrical tubes. Such techniques are frequently difficult to apply unless some rather simple models are introduced or some major simplifications are made. In order to reduce the difficulty encountered in such nonlinear creep studies, researchers frequently employ the concept of a reference stress at which the strain rate is insensitive to the creep power n (e.g., see Penny and Marriot [41]). In a probabilistic study of creep rupture Broberg [33] observed that in a $\ln \dot{\varepsilon}_c$ versus $\ln \sigma_o$ plot a variation in n leads to a pole at a certain stress, i.e., at the reference stress σ_R. Recently Boyle [42-44] has also developed a systematic method for the application of the reference stress technique in the study of creep rupture with random effects. In this section we will employ our inhomogeneous strain-dependent rupture

model to study the effect of random parameters on the rupture time for the case of a bar under constant tensile stress and a beam under pure bending.

5.a Random Inhomogeneous Creep Rupture for Uniaxial Constant Stress

We consider first the simple inhomogeneous constant stress case which was treated deterministically in 4(a). As we previously noted, the constant b in $H(x)$ [see Equation (69b)] may be considered a measure of the strength of a local imperfection. In this subsection, we shall simply assume that the randomness in the material occurs only through random fluctuations of b. Equation (69) with b a random variable is then written as

$$\hat{D}(x,t) = \frac{1}{Q}\,[a + \hat{b}\, e^{-c|x-x_o|}]\,\sigma_o^{\,n\alpha+\gamma}\,\hat{t}^{\,\alpha+\delta} \tag{99}$$

where the carat denotes a random quantity, and it is clear that now the damage is a random process. From Equation (99) we see at once that rupture will occur at position $x = x_o$ and the rupture time is the random variable

$$\hat{t}_R = \left[\tilde{D}_R\, Q\, \sigma_o^{-(n\alpha+\gamma)}\right]^{\frac{1}{\alpha+\delta}} (a+\hat{b})^{-\frac{1}{\alpha+\delta}} \tag{100}$$

Combining Equations (70,100) we have

$$\frac{\hat{t}_R}{t_{R_o}} = \left(\frac{a+\hat{b}}{H_o}\right)^{-\frac{1}{\alpha+\delta}} \tag{101}$$

where t_{R_o} and H_o are the deterministic values of $\hat{t}_R$ and $(a + \hat{b})$

for $\hat{b}$ equal to its mean value b_o. As previously noted, the experimental results indicate that b is always positive, and thus the random variable $\hat{b}'$ (defined as $\hat{b}' = \frac{\hat{b}}{b_o}$) will be distributed from zero to infinity. In order to evaluate the effect of randomness in $\hat{b}'$ on $\hat{t}_R$, we shall assume that $\hat{b}'$ is a two parameters log normal random variable [45], which distribution is frequently used to describe randomness in positive material constants (e.g., see [34]).

The first order density function of $\hat{b}'$ is accordingly given by

$$f_{\hat{b}'}(b') = \frac{1}{\sqrt{2\pi}\ \tau\ b'}\ e^{-\frac{(\ln b'-\eta)^2}{2\tau^2}}\ U(b') \tag{102}$$

The two parameters τ and η in Equation (102) specify the mean, variance and most probable value of $\hat{b}'$ in accordance with

$$E\{b'\} = \eta_{b'} = e^{\eta+\frac{1}{2}\tau} \tag{103a}$$

$$\tau^2_{b'} = e^{2\eta+\tau^2}(e^{\tau^2}-1) \tag{103b}$$

$$MAX\{\hat{b}'\} = e^{\eta-\tau^2} \tag{103c}$$

The first order density function of $\frac{\hat{t}_R}{t_{R_o}}$ (Equation (101)) is then obtained as (see [46])

$$f_{\hat{t}_R/t_{R_o}}\left(\frac{t_R}{t_{R_o}}\right) = \frac{H_o(\alpha+\delta)}{\sqrt{2\pi}\tau[(H_o t_R)^{-(\alpha+\delta)}-a](H_o t_R)^{1+\alpha+\delta}}\ e^{-\frac{\left\{\ln\frac{1}{b_o}[(H_o t_R)^{-(\alpha+\delta)}-a]-\eta\right\}^2}{2\tau^2}}\ U\left(\frac{t_R}{t_{R_o}}\right) \tag{104}$$

and the mean rupture time follows as

$$E\left\{\frac{\hat{t}_R}{t_{R_o}}\right\} = \frac{1}{H_o}\int_0^\infty (a+b_o b')^{-\frac{1}{\alpha+\delta}} \frac{1}{\sqrt{2\pi}\,\tau b'} e^{-\frac{(\ln b'-\eta)^2}{2\tau^2}} db' \qquad (105)$$

which in general must be evaluated numerically.

By using a Taylor series expansion about η_b, and neglecting higher order terms, we may obtain a simple closed form approximation for $E\left\{\frac{\hat{t}_R}{t_{R_o}}\right\}$ as

$$E\left\{\frac{\hat{t}_R}{t_{R_o}}\right\} \approx \frac{1}{H_o}\left\{\left(a+b_o e^{\eta+\frac{1}{2}\tau^2}\right)^{-\frac{1}{\alpha+\delta}} + b_o^2\left[\frac{1}{(\alpha+\delta)^2}+\frac{1}{\alpha+\delta}\right]\left(a+b_o e^{\eta+\frac{1}{2}\tau^2}\right)^{-\frac{1}{\alpha+\delta}-2} \times \frac{1}{2} e^{2\eta+\tau^2}\left(e^{\tau^2}-1\right)\right\} \qquad (106)$$

Another simple closed form approximation for $E\left\{\frac{\hat{t}_R}{t_{R_o}}\right\}$ may be obtained through the use of two-point estimates [47]. By this method the first order density function of $\hat{b}'$ is approximated by two delta functions, which have the same mean, second and third central moments as $\hat{b}'$. Accordingly, we obtain

$$E\left\{\frac{\hat{t}_R}{t_{R_o}}\right\} \approx \frac{1}{2H_o}\left\{\left[a+b_o(\eta_b+\tau_b)\right]^{-\frac{1}{\alpha+\delta}} + \left[a+b_o(\eta_b-\tau_b)\right]^{-\frac{1}{\alpha+\delta}}\right\} \qquad (107)$$

Table 1 gives a comparison of $E\left\{\frac{\hat{t}_R}{t_{R_o}}\right\}$ as obtained by numerical integration of Equation (105) and by approximate formulas (106,107) for AISI 310 stainless steel at 600°C, where $\alpha = 0.7$ and $\delta = 0.012$, and with $\tau = 0.5$. It shows that in this case these two closed form formulas do give a good estimate of $E\left\{\frac{\hat{t}_R}{t_{R_o}}\right\}$. Also, Figure 4 gives curves of $E\left\{\frac{\hat{t}_R}{t_{R_o}}\right\}$ plotted versus η for AISI 310 stainless steel at 600°C

with $\tau = 0.5$ and 0.8 as obtained by numerical integration of Equation (105). We see from these curves that, for a given value of $\hat{\tau}$, values of η in only a rather small interval yield values of $E\left\{\frac{\hat{t}_R}{t_{R_o}}\right\}$ in the vicinity of 1. In the next subsection we treat a more complex example, and also allow several parameters to be random variables.

5.b Random Creep Rupture in a Beam Under Pure Bending

Consider the same problem treated deterministically in 3(b), but now with a random inhomogeneous damage coefficient and random creep parameters. For the reasons previously discussed, we shall first rewrite Norton's law in terms of a reference stress σ_R in the form

$$\hat{\dot{\varepsilon}}_c = \hat{\dot{\varepsilon}}_R \left|\frac{\hat{\sigma}}{\sigma_R}\right|^{\hat{n}} \operatorname{sgn}(\hat{\sigma}) \tag{108}$$

where $\hat{\dot{\varepsilon}}_R$ and $\hat{n}$ are in general random variables, and where $\hat{\dot{\varepsilon}}_R$ is the strain rate at the reference stress σ_R. Note that at $\hat{\sigma} = \sigma_R$ the strain rate $\hat{\dot{\varepsilon}}$ is unaffected by random fluctuation in $\hat{n}$ (see Refs. [37] for a detailed discussion of Equation (108)). Proceeding as in the deterministic case we obtain governing equations

$$\hat{\dot{\varepsilon}}_c = y\,\hat{\dot{\kappa}} \tag{109a}$$

$$M = \sigma_R \left|\frac{\hat{\dot{\kappa}}}{\hat{\dot{\varepsilon}}_R}\right|^{\frac{1}{\hat{n}}} \hat{J} \operatorname{sgn}(\hat{\dot{\kappa}}) \tag{109b}$$

$$\hat{J} = \frac{2\hat{n}}{2\hat{n}+1} wh^2 h^{\frac{1}{\hat{n}}} \tag{109c}$$

$$\vartheta = \frac{M|y|^{\frac{1}{\hat{n}}}}{\hat{J}} \operatorname{sgn}(y) \tag{109d}$$

$$\hat{\sigma}_{max} = \frac{2\hat{n}+1}{2\hat{n}} \frac{M}{wh^2} \tag{109e}$$

Also, the damage law with $D_o(x) = 0$ will be taken as

$$\tilde{D}(x,t) = \hat{C}(x)\ \dot{\hat{\varepsilon}}^{\alpha}(x,t)\ \hat{\sigma}_{max}{}^{\gamma}\hat{t}^{\delta} \tag{110}$$

where $\hat{C}(x)$ is a random process in x .

We shall assume that $\hat{C}(x)$ takes the similar form as Equation (69b), i.e.,

$$\hat{C}(x) = \hat{a} + \hat{b}\ e^{-c|x-x_o|} \tag{111}$$

but now both $\hat{a}$ and $\hat{b}$ may be random variables due to the material imperfections. Then the damage is a random process, expressed in terms of four random variable material parameters, obtained by substituting Equations (108,111) into Equation (110), giving

$$\tilde{D}(x,t) = (\hat{a} + \hat{b}\ e^{-c|x-x_o|})\dot{\hat{\varepsilon}}_R{}^{\alpha}\sigma_R{}^{\gamma}\left|\frac{\sigma_{max}}{\sigma_R}\right|^{\alpha\hat{n}+\gamma}\hat{t}^{\alpha+\delta} \tag{112}$$

Although there are several expressions for the reference stress σ_R given in [43], they differ very little from the simplest expression $\sigma_R = \frac{M}{wh^2}$ for the interval $3 \leq n \leq 11$. Therefore, for convenience we choose $\sigma_R = \frac{M}{wh^2}$ in Equation (112) and obtain a simpler form for the damage law as

$$\tilde{D}(x,t) = (\hat{a} + \hat{b}\ e^{-c|x-x_o|})\dot{\hat{\varepsilon}}_R{}^{\alpha}\sigma_R{}^{\gamma}\left(\frac{2\hat{n}+1}{2\hat{n}}\right)^{\alpha\hat{n}+\gamma}\hat{t}^{\alpha+\delta} \tag{113}$$

From Equation (113), it is clear that rupture will occur at $x = x_o$ (if $D_o(x) = 0$), and the initial rupture time (i.e., the time at which the beam begins to rupture) is the random variable

$$t_{R1} = \left[\frac{D_R}{(\hat{a} + \hat{b})\dot{\hat{\varepsilon}}_R{}^{\alpha}\sigma_R{}^{\gamma}}\left(\frac{2\hat{n}+1}{2\hat{n}}\right)^{-(\alpha\hat{n}+\gamma)}\right]^{\frac{1}{\alpha+\delta}} \tag{114}$$

Assuming that $\hat{a}$, $\hat{b}$, $\dot{\hat{\varepsilon}}_R$ and n are independent random variables,

we may find an expression for the mean of the initial rupture time as

$$E\left\{\frac{\hat{t}_{RI}}{t_{RI_o}}\right\} = \frac{1}{B_o}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{1}{a+b}\right)^{\frac{1}{\alpha+\delta}} f_{\hat{a}}(a)\, f_{\hat{b}}(b)\, da\,db$$

$$\times \int_{-\infty}^{\infty}\left(\frac{1}{\dot{\varepsilon}_R}\right)^{\frac{\alpha}{\alpha+\delta}} f_{\hat{\dot{\varepsilon}}_R}(\dot{\varepsilon}_R)\, d\dot{\varepsilon}_R \int_{\infty}^{\infty}\left(\frac{2n+1}{2n}\right)^{-\frac{\alpha n+\gamma}{\alpha+\delta}} f_{\hat{n}}(n)\, dn \qquad (115)$$

where $B_o = \left(\frac{1}{a_o+b_o}\right)^{\frac{1}{\alpha+\delta}}\left(\frac{1}{\dot{\varepsilon}_R}\right)^{\frac{\alpha}{\alpha+\delta}}\left(\frac{2n_o+1}{2n_o}\right)^{-\frac{\alpha n_o+\gamma}{\alpha+\delta}}$, and t_{RI_o} is the deterministic value of $\hat{t}_{RI}$ when $\hat{a}$, $\hat{b}$, $\hat{\dot{\varepsilon}}_R$ and $\hat{n}$ are equal to their mean values a_o, b_o, $\dot{\varepsilon}_{R_o}$ and n_o.

As in last section, we may approximate the mean of $\frac{\hat{t}_{RI}}{t_{RI_o}}$ by a Taylor series expansion, yielding

$$E\left\{\frac{\hat{t}_{RI}}{t_{RI_o}}\right\} \approx \frac{1}{B_o}\left\{(\eta_a+\eta_b)^{-\frac{1}{\alpha+\delta}} + \frac{1}{2}\left[\frac{1}{(\alpha+\delta)^2}+\frac{1}{\alpha+\delta}\right](\eta_a+\eta_b)^{-\left(\frac{1}{\alpha+\delta}+2\right)}(\tau_a^2+\tau_b^2)\right\} \times \left\{(\eta_{\dot{\varepsilon}_R})^{-\frac{\alpha}{\alpha+\delta}} + \frac{1}{2}\left[\left(\frac{\alpha}{\alpha+\delta}\right)^2+\frac{\alpha}{\alpha+\delta}\right](\eta_{\dot{\varepsilon}_R})^{-\left(\frac{\alpha}{\alpha+\delta}+2\right)}\tau_{\dot{\varepsilon}_R}^{2}\right\}$$

$$\times \left\{\left(\frac{2\eta_n+1}{2\eta_n}\right)^{-\frac{\alpha\eta_n+\gamma}{\alpha+\delta}} + \frac{1}{2}\frac{d^2}{dn^2}\left[\left(\frac{2n+1}{2n}\right)^{-\frac{n\alpha+\gamma}{\alpha+\delta}}\right]\Bigg|_{n=\eta_n}\tau_n^2\right\} \qquad (116)$$

We may also use the two-point estimate technique to obtain an alternative simple approximate expression for $E\left\{\frac{\hat{t}_{RI}}{t_{RI_o}}\right\}$ as

$$E\left\{\frac{\hat{t}_{RI}}{t_{RI_o}}\right\} \approx \frac{1}{B_o}\,\frac{1}{4}\Big\{[(\eta_a+\tau_a)+(\eta_b+\tau_b)]^{-\frac{1}{\alpha+\delta}} + [(\eta_a+\tau_a)+(\eta_b-\tau_b)]^{-\frac{1}{\alpha+\delta}}$$

$$+[(\eta_a-\tau_a)+(\eta_b+\tau_b)]^{-\frac{1}{\alpha+\delta}} + [(\eta_a-\tau_a)+(\eta_b-\tau_b)]^{-\frac{1}{\alpha+\delta}}\Big\}$$

$$\times \frac{1}{2}\left[(\eta_{\dot{\varepsilon}_R} + \tau_{\dot{\varepsilon}_R})^{-\frac{\alpha}{\alpha+\delta}} + (\eta_{\dot{\varepsilon}_R} - \tau_{\dot{\varepsilon}_R})^{-\frac{\alpha}{\alpha+\delta}}\right]$$

$$\times \frac{1}{2}\left\{\left[\frac{2(\eta_n+\tau_n)+1}{2(\eta_n+\tau_n)}\right]^{-\frac{\alpha(\eta_n+\tau_n)+\gamma}{\alpha+\delta}} + \left[\frac{2(\eta_n-\tau_n)+1}{2(\eta_n-\tau_n)}\right]^{-\frac{\alpha(\eta_n-\tau_n)+\gamma}{\alpha+\delta}}\right\} \tag{117}$$

Given the properties of all the random variables, one may now evaluate $E\left\{\frac{\hat{t}_{RI}}{t_{RI_o}}\right\}$ either by a numerical integration of Equation (115) or approximately by means of the closed form formulas (116,117).

6. BRIEF SUMMARY

The continuum mechanical and the physical metallurgical approaches employed in various previous deterministic, homogeneous, strain-independent, uniaxial creep rupture theories were first discussed in some detail. Previous multiaxial creep rupture theories were also briefly discussed, with the damage treated either as a scalar or as a tensor quantity. Then the deterministic, homogeneous, uniaxial tensile, strain-dependent creep damage theory developed by Belloni, Bernasconi, Cozzarelli and Piatti from both the continuum mechanical and physical metallurgical approaches was studied in considerable detail.

Based on previous experimental results obtained at J.R.C. Ispra it was postulated that no damage is produced by compression, and using this postulation the Belloni et al. strain-dependent creep damage theory was generalized to include compressive stress. A multiaxial strain-dependent creep damage theory was also developed by a simple extension of the uniaxial theory with the damage treated as a scalar. The problem of a beam under pure bending was then studied using this strain-dependent creep damage theory. At the instant of initial rupture a failure front is formed at the top outside fiber of the beam, and then this failure front propagates through the entire cross-section, resulting in a complete rupture of the beam.

An analysis of creep rupture data from J.R.C. Ispra for specimens of various lengths at constant temperature and under constant tensile stress resulted in the postulation of a local inhomogeneous

damage law. It was shown that the combined material parameter reached a peak of a certain magnitude superposed on a constant value; the magnitude of this peak was interpreted as a measure of the strength of local imperfection. From this local inhomogeneous damage law, the local critical value of damage was shown to be significantly closer to the theoretical limiting value of 1 than the average value. The local inhomogeneous damage law was also generalized to include both tensile and compressive stress. Then using this local inhomogeneous damage law, the problem of inhomogeneous creep rupture under constant load for both large and small deformation was reexamined, thereby extending previous results based on homogeneous theory.

Also, the inhomogeneous strain-dependent creep rupture model was used to examine the effect of random parameters on rupture time using classical probability techniques. In the case of a bar under constant tensile stress, the magnitude of the peak of the combined material parameter was treated as a random variable. The mean value of the rupture time was then obtained from numerical integration and from two simple closed form approximations which were in good agreement with the results of the numerical integration. This mean value demonstrated that the rupture time is highly dependent on random imperfection variations. Finally, the beam under pure bending with several parameters chosen as random variables was reexamined using a similar approach.

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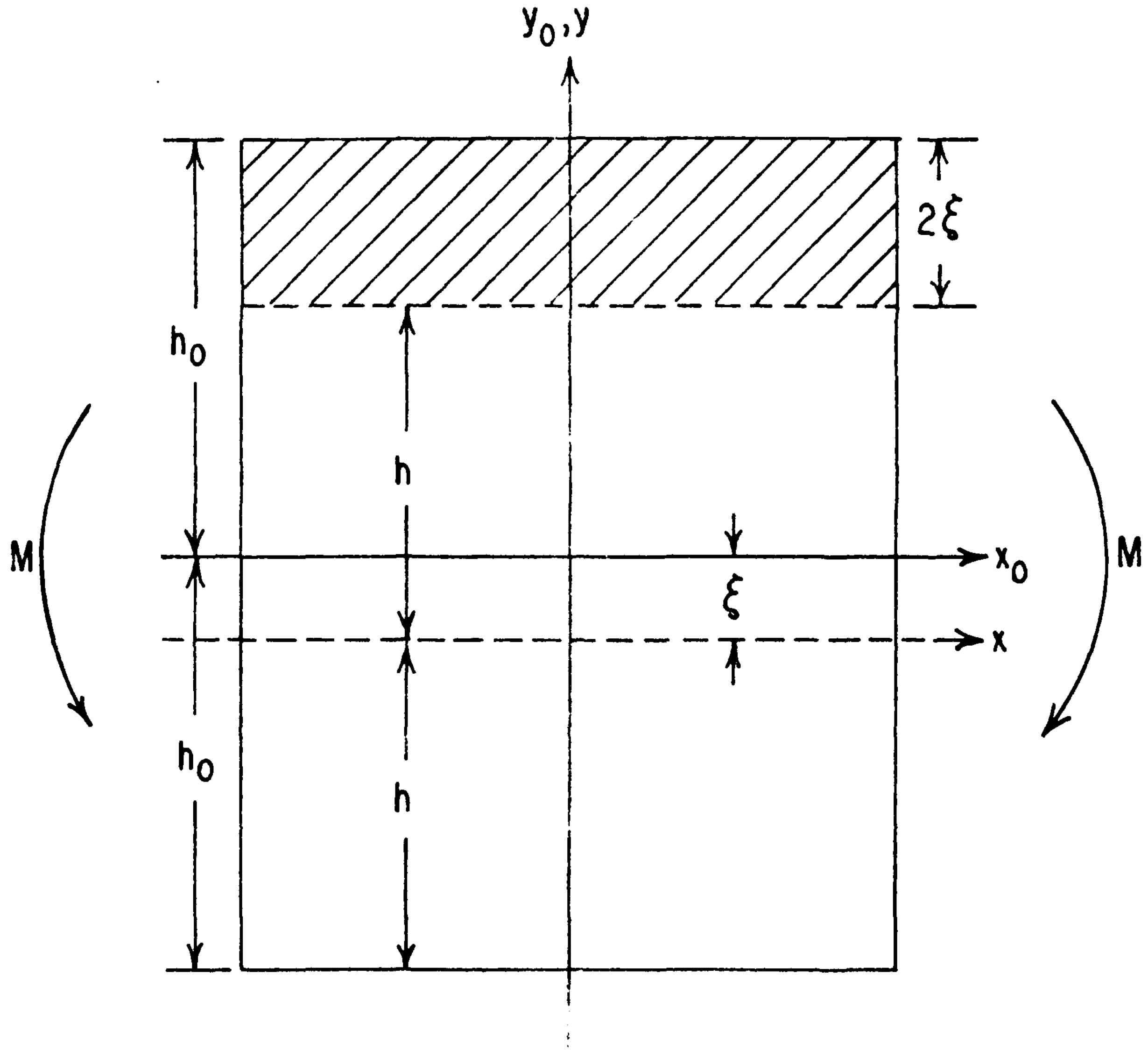

Figure 1. Geometry of Beam Under Pure Bending

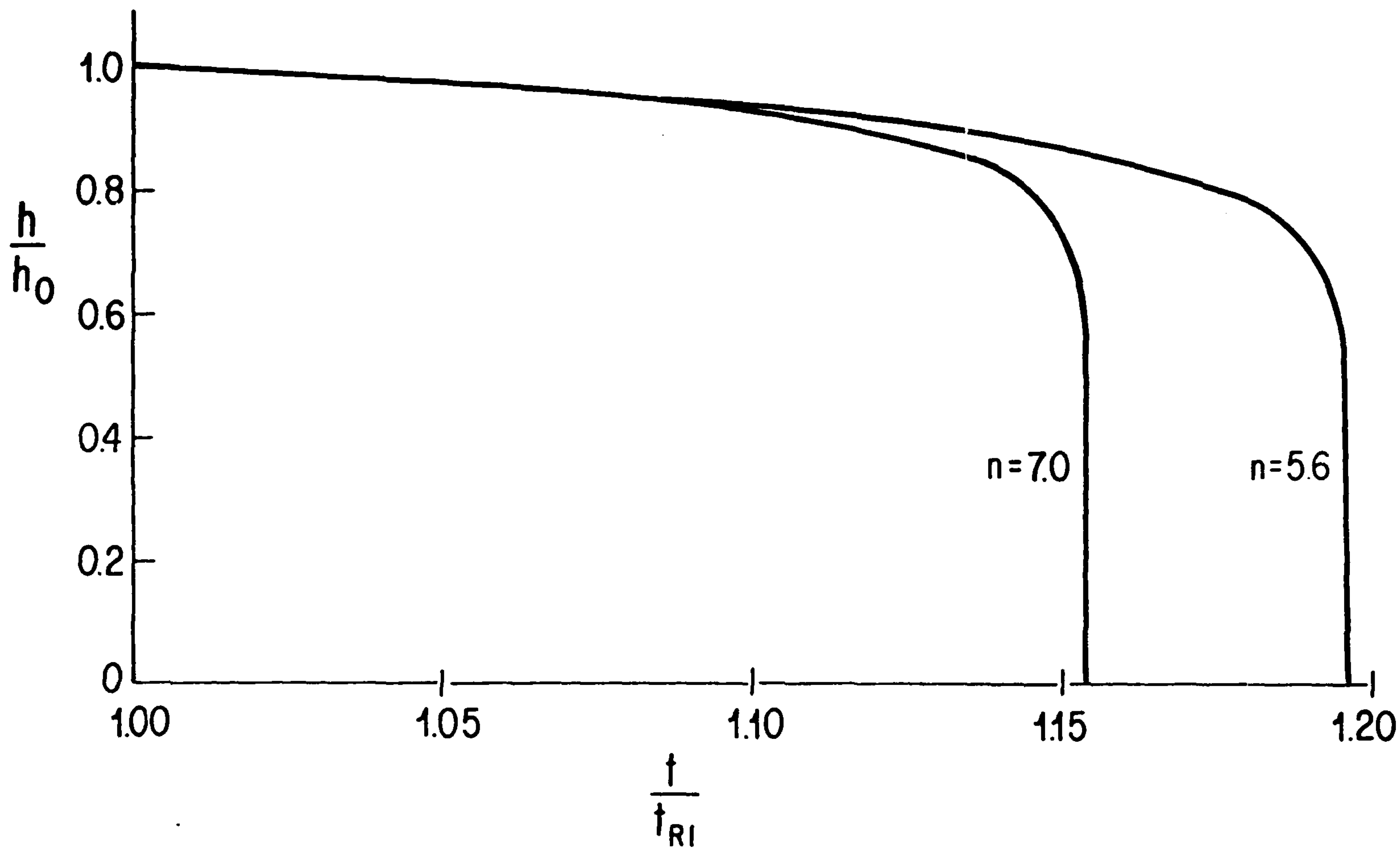

Figure 2. Effective Depth of Beam h/h_O vs. Time t/t_{RI}

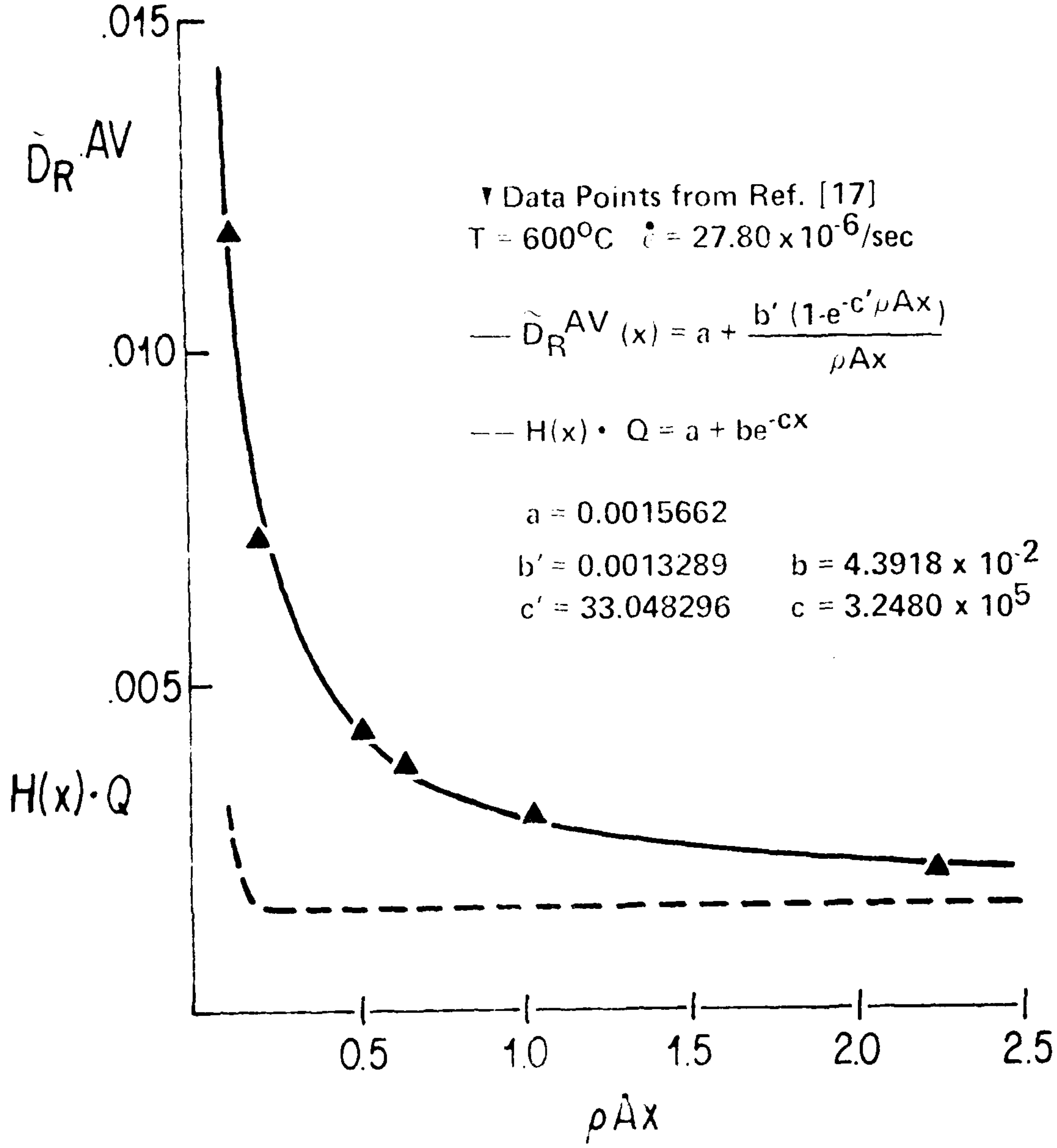

Figure 3. Average Damage at Rupture $\dot{D}_R^{AV}$ and Combined Material Parameter $H(x)\cdot Q$ vs. Mass ρAx

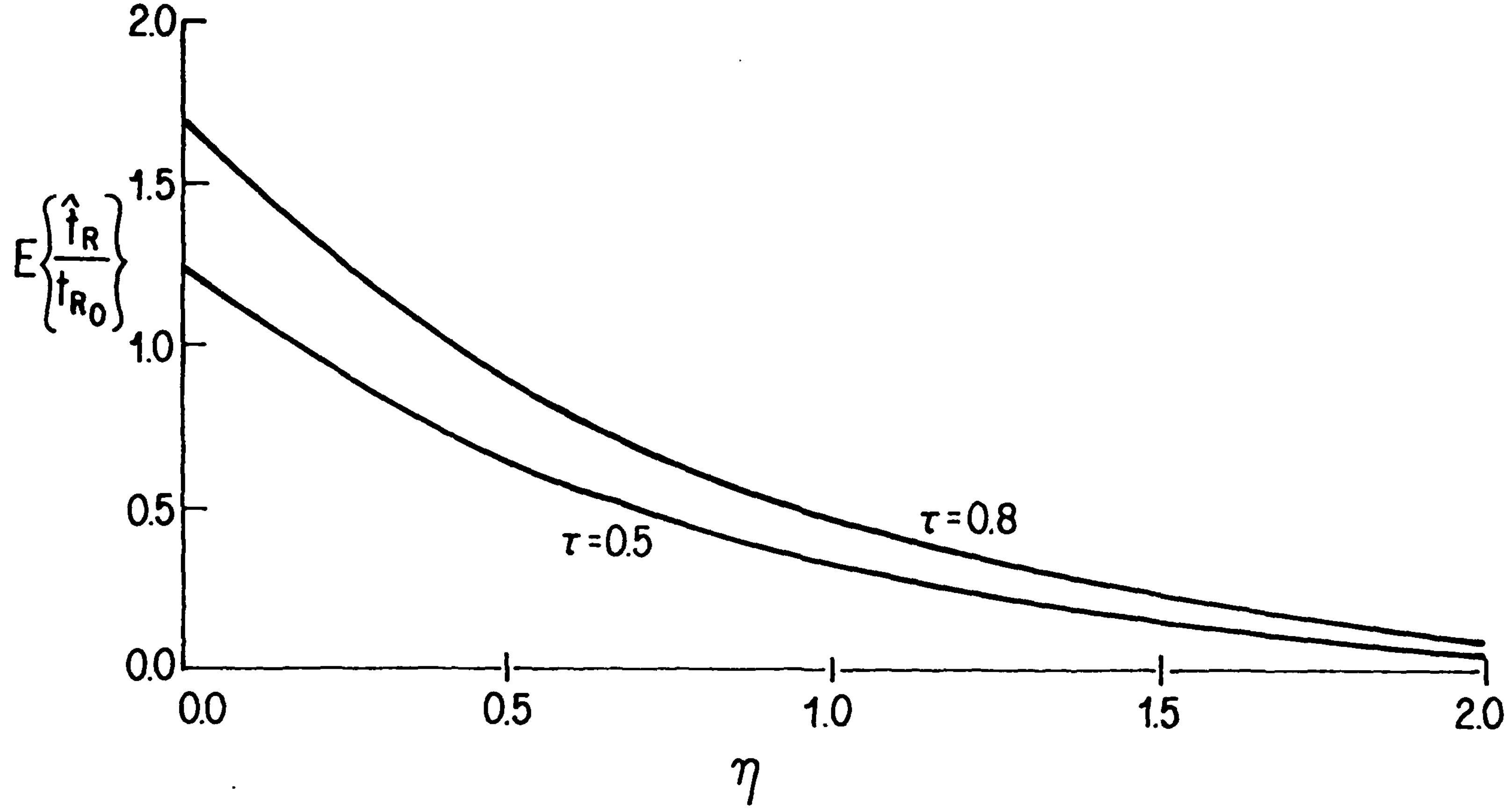

Figure 4. Mean of $\hat{t}_R / t_{R_0}$ vs. η, for $\tau = 0.5, 0.8$

TABLE 1 - Mean of $\frac{\hat{t}_R}{t_{R_o}}$, $E\left\{\frac{\hat{t}_R}{t_{R_o}}\right\}$

$\xi = 0.5$

	AISI 310 Stainless Steel at 600°C		
n	Numerical Integration	Taylor Series	Two-Point Estimates
0.5	0.63507	0.48712	0.53285
1.0	0.32050	0.24443	0.26800
1.5	0.16061	0.12205	0.13401
2.0	0.08013	0.06075	0.06677

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Employing a recent homogeneous strain-dependent creep damage theory, the propagation of a failure front in a beam under pure bending is studied. A local inhomogeneous strain-dependent creep damage theory is postulated, based on creep damage data obtained at J.R.C. Ispra. Solutions for various loading conditions are obtained, and are shown to be in better agreement with observation than previous theories. The effect of random material parameters on the rupture time is considered for the constant tensile stress test and for the beam under pure bending.

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